Statistical Methods and Quality Control[1]

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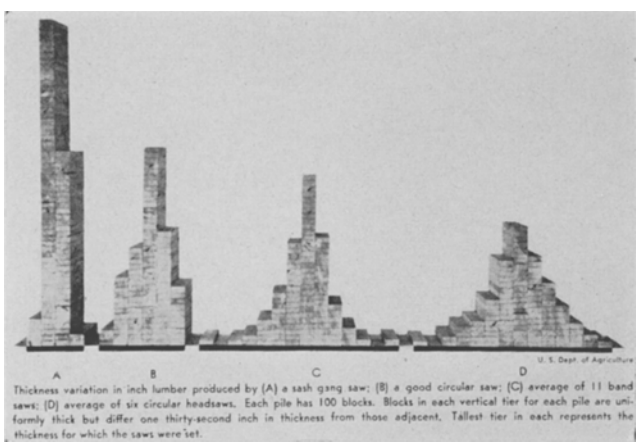
STATISTICS, as the pronunciation of its name does not indicate, started as the collection of information about the state, births, deaths, etc. It should properly be called "State"istics. Besides the name there is another hangover from the original object of inquiry. The term population is used occasionally to describe the complete collection of men, women, cottonseed, or brick which is to be sampled and judged. These remarks are directed to leading up to and amplifying the A.S.T.M. Manual on the Quality Control of Materials.

Figure 1 shows boards cut by various types of saws, 100 blocks as shown for each type of saw. A, the gang saw, has most of its production in two groups,



Thickness variation in inch lumber produced by (A) a sash gang saw; (B) a good circular saw; (C) average of 11 band saws; (D) average of six circular headsaws. Each pile has 100 blocks. Blocks in each vertical tier for each pile are uniformly thick but differ one thirty-second inch in thickness from those adjacent. Tallest tier in each represents the thickness for which the saws were set.

FIG. 1

but if one had a thousand blocks from A instead of 100, the tendency to fan out would be observed. The bull's eye for each saw was 1 in. The saws were adjusted so that on the average they produced 1-in. boards. It is probably fair to say that each saw operation had its process well under control.

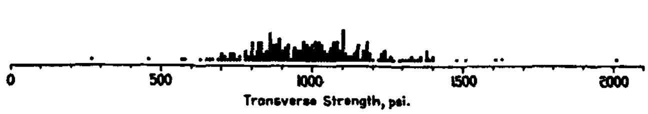


FIG. 2

A similar set of observations is shown in Figure 2. This is from the A.S.T.M. Manual and shows transverse breaking strength of bricks. The reasons for this shape are evident if we examine in detail the things that take place in a test or series of tests. For simplicity, let us consider the case where a number of analyses are made on replicate samples. If a liquid is chosen for analysis and the sample is well shaken, the assumption that the samples are all alike is fairly true. A good example of this type is the analysis of glycerin by the bichromate oxidation method.

In this test we have a fairly large number of points at which measurement is made. The final result has an "error" which is the algebraic sum of all the errors made. The various points at which variations or "errors" in measurement can be made are: the standardizing of the bichromate solution; the weighing of the sample of glycerin; the transference of the glycerin to the flask; the making up to volume; the uniformity of the solution of glycerin attained by shaking; the variations in pipetting; the variations in the completeness of the oxidation; the error in back titrating as to a) burette reading and b) as to end point.

Suppose in a determination like this, we have eight points at which variation can occur; suppose (for the sake of simplifying the discussion) the variations are all of the same size and equal 0.1% glycerin. If we made the determination 256 times, we would have the deviations piling up like Figure 3.[2]

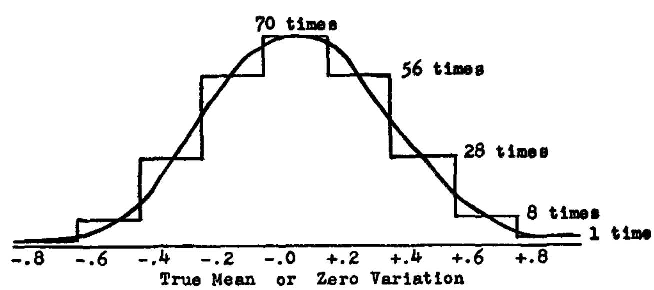


FIG. 3

In an actual system the variations are not of equal size, nor do they change abruptly from the +0.1 to the −0.1 value. The shape of the distribution therefore smooths out and becomes somewhat like the line in the previous figure.

The stepped polygon, representing the "heads and tails" distribution, is the binomial expansion $(a+b)^8$ where the height of the steps is given by the coefficients of the terms of the expansion. As the exponent of the expansion becomes larger and larger, the steps become smaller and smaller. To fit the general case for all expansions, De Moivre developed in 1733 the "probability" curve $y = ke^{-h^2x^2}$. Before assigning more useful values to the constants "k" and "h," we want to consider various methods of expressing the "central" value in a distribution and of expressing the dispersion or amount of scatter.

Refer to the breaking strength of brick. Ordinarily there is no point in having the scale spread out so that each test will be charted to the full scale of its original value. The strength of bricks was determined to 10 lbs. To show each at its original value required

[1] The figures are all from the A.S.T.M. Manual on the Quality Control of Materials except No. 1 from the U. S. Department of Agriculture; No. 3, 7, and 8 compiled by the author; and No. 9 from the book "Economic Control of Quality of Manufacture Product" by W. A. Shewhart.

[2] These values are based on probability mathematics as applied to "heads and tails" for 8 coins. The +0.1 or the −0.1 value for each cause of variation is considered to occur just like heads or tails.

$\frac{2010-270}{10}$ or 174 divisions. The work can be simplified and the scales compressed by plotting "grouped" frequencies. Figure 4 shows that data on the bricks plotted "grouped." The number of "cells" in a grouped distribution should preferably be between 11 and 20.

Transverse Strength, psi.		Frequency
225 to 375	|	1
375 to 525	|	1
525 to 675	卌 |	6
675 to 825	卌 卌 卌 卌 卌 卌 卌 |||	38
825 to 975	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌	80
975 to 1125	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 |||	83
1125 to 1275	卌 卌 卌 卌 卌 卌 卌 ||||	39
1275 to 1425	卌 卌 卌 ||	17
1425 to 1575	||	2
1575 to 1725	||	2
1725 to 1875		0
1875 to 2025	|	1
	Total	270

FIG. 4

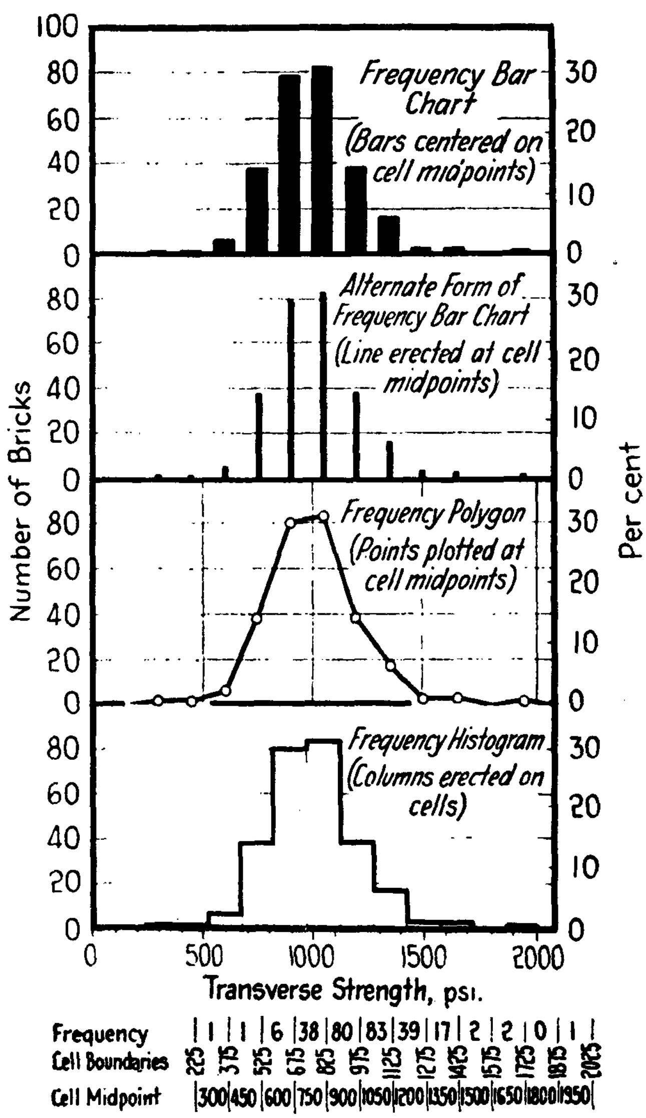


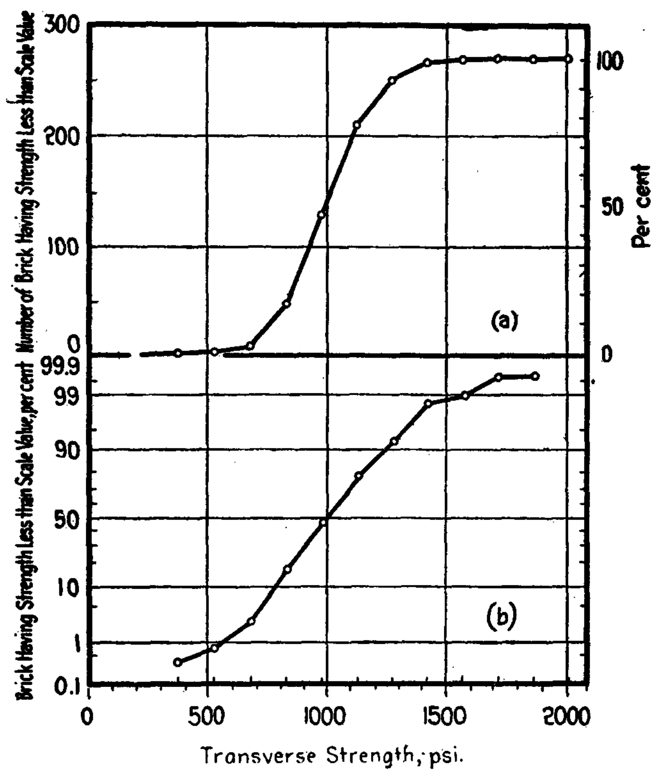
FIG. 5

If the number of observations is small, 25 to 250, as few as 10 cells may be used. Below 25 observations there is not much point in grouping.

Figure 5 shows graphical presentations of the same data.

Cumulative Frequency

For some purposes, particularly age test or "life" test, a cumulative grouping is desirable. The values may be plotted on a "less than" or greater than basis. The values for the bricks are collected in this fashion on a "less than" basis in the upper part of Figure 6. In the lower part of Figure 6 they are plotted



(a) Using arithmetic scale for frequency.
(b) Using probability scale for relative frequency.

FIG. 6

percentagewise on probability paper. This paper is designed to give a straight line to a normal cumulative or "ogive" curve.

Using the Normal Distribution Functions

Two attributes of a group of data, collected and plotted in the fashion shown, are immediately apparent. Some measure of central value is useful, and some way of expressing the scatteration about this central value would be of value. Three values of central values are used; the choice is made according to the purpose for which it is used.

The mean or average value is, of course, the most common and most useful. The median or middlemost value has definite utility as a preliminary value. If the observations are tallied up, the median can be determined by counting. The mode or peak value has special utility in a "skewed" distribution. Skewed distribution will be discussed later.

The scatteration can be expressed in several ways. It is nearly always expressed as some function of the distances of the points from the mean value.

The most obvious measure is the average deviation. The differences between the mean value and the individual values are summed up without regard to + or — signs and divided by the number of values. This value is not normally used; first, because it is not the most efficient measure of scatter, and, second, because it is tedious to calculate.

The (so-called) standard deviation is the commonly used measure of scatteration. It is more properly called the root-mean-square deviation. The standard deviation or σ is obtained by determining the differences between each value (X) and the mean (M). Each difference is squared. The squares are summed, the sum is divided by n (the number of observations), and the square root of the quotient is taken.

In algebraic form: $\sigma = \sqrt{\frac{\Sigma(X-M)^2}{n}}$.

In practice this is simplified as follows:

$$\frac{\Sigma(X-M)^2}{n} = \frac{\Sigma X^2 - 2M\Sigma X + M^2}{n}$$

but $\Sigma X = nM$ and $\Sigma M^2 = nM^2$ so $\Sigma X^2 - 2M\Sigma X + \Sigma M^2 + \Sigma X^2 - 2nM^2 + nM^2$ so $\frac{\Sigma(X-M)^2}{n} = \frac{\Sigma X^2}{n} - \frac{nM^2}{n}$ or $\sigma = \sqrt{\frac{\Sigma(X-nM)^2}{n}} = \sqrt{\frac{\Sigma X^2}{n} - M^2}$.

This may be written $\sqrt{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2}$.

σ may then be computed by summing up the squares of the numbers and the first powers of the numbers. A simple manipulation gives σ. The amount of work can be further simplified by grouping and coding.

Skewed Distributions

One of the most common type of skewed distributions is encountered where the values approach 0 (or 100%). Even by analytical error it is hard to get a value below 0 (or over 100%).

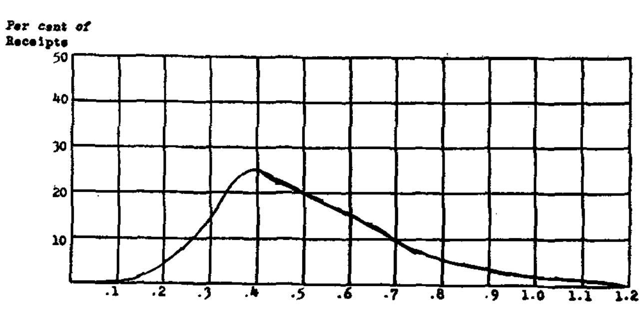


FIG. 7

Figure 7 shows the percentage of unsaponifiable in tallow received at a factory. More than 10 million pounds are represented. The values spread from 0 to 1.2 with the peak at about 0.42. The mean and the standard deviation do not completely define the characteristics of such a collection of data. A third parameter skewness is required. A mathematical measure of this can be derived from the summation of the cubes of the deviations from the mean in a manner somewhat similar to the manner in which the standard deviation is derived from the squares of the deviations from the mean. These three parameters are difficult to handle. It is much simpler to plot the values cumulatively on probability-log paper, as in Figure 8. This enables one to draw conclusions from the data quickly and easily.

Grouped Frequency Method for M and σ

Referring back to Figure 4, column f is frequency. The values for strength and frequency are copied below in a reverse order to conform to the convention that as one goes up from the origin the values increase.

	f	d	fd	fd^2	
1875 - 2025	1	+6	+ 6	36	
1725 - 1875	0	+5	0	0	
1575 - 1725	2	+4	+ 8	32	
1425 - 1575	2	+3	+ 6	18	
1275 - 1425	17	+2	+34	68	
1125 - 1275	39	+1	+39	39	
975 - 1125					1050
825 - 975	80	−1	−80	80	
675 - 825	38	−2	−76	152	
525 - 675	6	−3	−18	54	
375 - 525	1	−4	− 4	16	
225 - 375	1	−5	− 5	25	
	n = 270		Σ + 93	520	

$\frac{\Sigma fd}{n} = \frac{-90}{270} = -.333$ Σ − 183

$\frac{\Sigma fd^2}{n} = \frac{520}{270} = 1.926$ Σ − 90

M = 1050 − 150(.333) = 1000

$S = i\sqrt{1.926 - (-.333)^2} = 150$ $\sqrt{\begin{array}{r}1.926\\ .111\\ \hline 1.815\end{array}}$

(σ) S = 150 × 1.347 = 202.1

Column d is the distance, in cell units, from an assumed origin. The assumed origin may be placed wherever convenient. The writer prefers to place it where it will minimize the size of the numbers to be handled. In the tabulation it is midpoint of the 975 to 1,125 cell or 1,050. It will be observed that this is a method of "coding" the values. In effect, 1,050 has been subtracted from each value and the result divided by 150. Column fd is thus ΣX. The values in fd are multiplied by d giving fd^2. This is obviously ΣX^2. Each value is divided by n giving us $\frac{\Sigma X^2}{n}$ and $\frac{\Sigma X}{n}$ These are the values basic to the calculation of σ. Before calculating σ, we should use the $\frac{\Sigma X}{n}$ or mean value (coded) to calculate the value of the mean in the original numbers

$$\frac{\Sigma fd}{n} = -.333$$

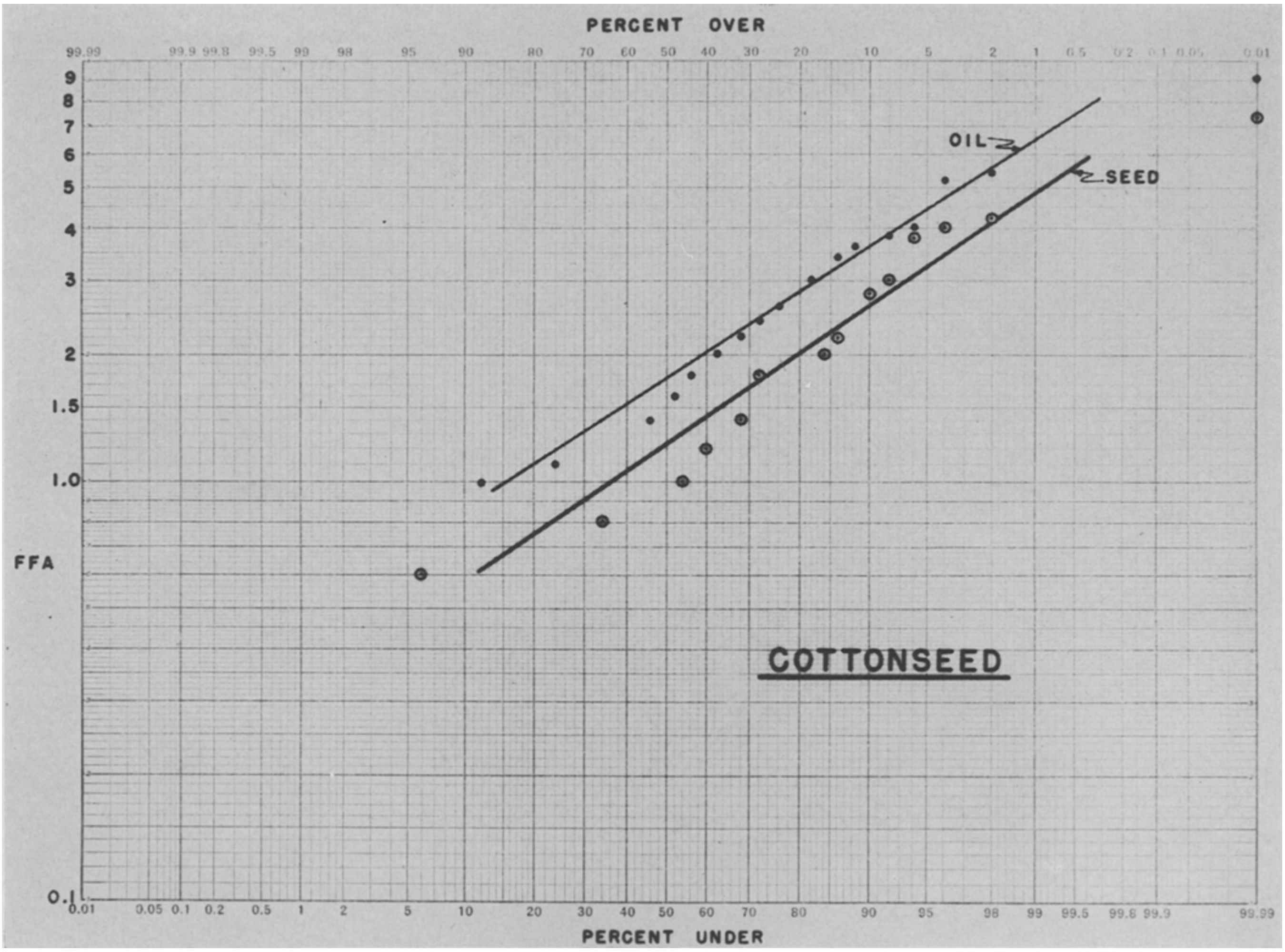


FIG. 8

Hence the mean = 1,050 — 150 × .333 or 1,000. The value of the class interval (150) may be denoted by i (for interval). Likewise a definite group of numbers is frequently denoted by S. Properly speaking, σ is reserved for all of the bricks (or other units) produced by the process. The value S is used for the standard deviation of a sample or small portion of the whole production.

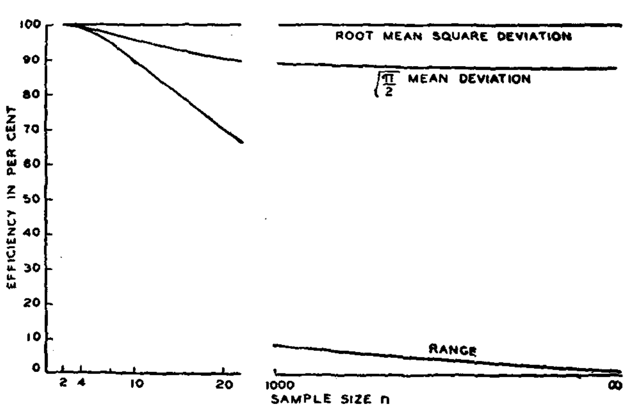


FIG 9.—EFFICIENCY OF $\sqrt{\frac{\pi}{2}}$ TIMES MEAN DEVIATION AND RANGE AS ESTIMATES OF σ COMPARED WITH THAT OF THE STANDARD DEVIATION.

$$\text{Then: } S = i\sqrt{1.926 - (-.333)^2}$$

$$150\sqrt{1.815} = 202.1$$

For small samples (under 30) the standard deviation is better represented by $\sqrt{\frac{\Sigma d^2}{n-1}}$. The S computed by the use of n in the denominator can be corrected by multiplying by $\sqrt{\frac{n}{n-1}}$. Graphs or tables of this are available.

Table 9 shows the reason for picking the standard deviation. It also shows why the range can be used as a measure of scatteration for small samples. It is

Lot	Sample Size, n	Average, $\bar{X}$	Range, R
No. 1	5	36.0	6.6
No. 2	5	31.4	0.5
No. 3	5	39.0	15.1
No. 4	5	35.6	8.8
No. 5	5	38.8	2.2
No. 6	5	41.6	3.5
No. 7	5	36.2	9.6
No. 8	5	38.0	9.0
No. 9	5	31.4	20.6
No. 10	5	29.2	21.7

FIG. 10

Central Lines
For $\bar{X}$: $\bar{X}' = 35.00$.
$n = 5$:
For R: $d_2\sigma' = 2.326\ (4.20) = 9.8$.

Control Limits
$n = 5$:
For $\bar{X}$: $\bar{X}' \pm A\sigma' = 35.00 \pm (1.342)\ (4.20)$, 40.6 and 29.4.
For R: $D_2\sigma'$ and $D_1\sigma'$ = (4.918) (4.20) and (0) (4.20), 20.7 and 0.

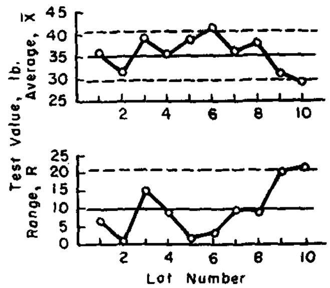


FIG. 11.—Control Charts for $\bar{X}$ and R.
Small samples of equal size, $n = 5$; X', σ' given

RESULTS.—Lack of control is indicated by results for lots Nos. 6 and 10. Corrective action is required both with respect to averages and with respect to variability within a lot.

not that the range is so good, it is that the standard deviation is so poor for small samples.

The Control Chart method of Quality Control is a method of keeping track of the variations in the mean value and the degree of scatter from the mean. The procedure is to take samples of relatively small size—from 4 to 10 usually—and determine the mean and the range on each sample. The samples should be of the same size, and since the size is small, the range is a good measure of scatter. Values for a group of such samples are given in Figure 10.

In setting up the graphs upon which such values are plotted, it is very helpful if the scatter about the mean value has been determined previously. One can then enter the tables provided in the manual and draw charts as shown in Figure 11.

By observing the location of the points on such charts, one can determine whether the operation is aimed at the right mean value and whether the deviations from this value are under control. Since the limits are 3σ limits, the chances of a properly handled sample being out of limits by sheer chance are only 3 in 1,000. For this reason the out-of-limit values are an indication of lack of control.

Basic Theory of Automatic Control

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DURING the past half century the chemical industry in the United States has been one of the nation's prime examples of the continuing evolution of industrial technology. New processes have been introduced, new products have been produced, and, perhaps even more significant, ever greater manufacturing efficiencies have been attained. Sometimes when we look back that far, statistics get a little fuzzy. In the case of the chemical industry the changes and developments since the turn of the century have been so revolutionary that statistics back that far would be virtually meaningless. Let's look at just the last 15 years. This period in itself presents a revealing picture.


C. W. Bowden Jr.

The volume of production in the chemical industry in the year ending April 1953 totaled more than $19 billion, as contrasted with less than $4 billion in 1939. This is a 190% increase in volume. If we adjust for price changes, etc., it means that today's physical volume is three times what it was 15 years ago.

Obviously this hasn't been accomplished simply by the influx of hordes of additional workers. Actually, if we go back to our statistics, we see that employment is less than twice that of 1939.

The chemical industry has reached this volume by encouraging and accepting technological advances, by introducing new continuous methods of production, and by the continued application of more and more automatic instruments to help control these operations. With the new technology today's chemical plant worker turns out some $26,000 worth of material while his 1939 counterpart could produce only $15,000 worth. This is a pretty impressive growth picture, is it not—to have happened in only 15 years?

The oil and fat industry can take a bow also for its segment of the chemical industry has had parallel progress. Not to go into another statistical study, there is one basic comparison which fully illustrates the growth of the soaps, fats, and oils industry. In prewar days the United States annually imported some 1.3 billion pounds of fats and oils. Today the industry's progress is reflected in the fact that the United States is a net *exporter* of 1.1 billion pounds.

This change has resulted from a number of factors. There has been the increase in production of domestic oil-bearing materials and the effect of wartime dislocations. But one of the major factors has been the continuing development of solvent extraction methods. Today's techniques harness a multiplicity of industrial instruments to control these processes.

For example, at the Glidden plant in Indianapolis the processing of soybeans is accomplished by matching improvements in equipment with the application of new control techniques. These recording and recording-controlling instruments made by Honeywell maintain close watch over the process variables. The instruments thus contribute efficient operation of the continuous extractor, eliminating the hazards and expense of solvent loss while increasing the yield of high-quality, reproducible end-products.

We all realize that this rapid change-over to continuous operation has not been confined to any particular branch of the chemical industry. It has been industry-wide and is being further accelerated by economic conditions. The large growth of markets, the scarcity of qualified operators, and the rising labor costs have compelled all producers to seek maxi-